Online Appendix to the paper "The Structural Transformation Between Manufacturing and Services and the Decline in the U.S. GDP Volatility"

Alessio Moro*

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*Department of Economics, University of Cagliari, amoro@unica.it.

Appendix D: The Relative Price p_s/p_m and the Aggregate Production Functions

The marginal rate of transformation ϕ appearing in the planner's problem (20) in Appendix B corresponds to the relative price of the two goods in the market economy, p_s/p_m. To obtain this relative price, it is convenient to exploit the Cobb-Douglas form of the production function to derive the *net production function* of each sector, defined as the amount of gross output of a sector minus the intermediate goods produced and used in the same sector. The net production function for manufacturing is obtained by solving the following problem

$$Y_m = \max_{M_m} \left\{ B_m \left(K_m^{\alpha} N_m^{1-\alpha} \right)^{\nu_m} \left(M_m^{\varepsilon_m} S_m^{1-\varepsilon_m} \right)^{1-\nu_m} - M_m \right\} \tag{1}$$

and it is equal to

$$Y_m = \Phi_{m1} B_m^{\frac{1}{1-\varepsilon_m(1-\nu_m)}} K_m^{\frac{\alpha\nu_m}{1-\varepsilon_m(1-\nu_m)}} N_m^{\frac{(1-\alpha)\nu_m}{1-\varepsilon_m(1-\nu_m)}} S_m^{\frac{(1-\varepsilon_m)(1-\nu_m)}{1-\varepsilon_m(1-\nu_m)}}, \tag{2}$$

where $\Phi_{m1} = \left[1 - \varepsilon_m (1-\nu_m)\right] \left[\varepsilon_m (1-\nu_m)\right]^{\frac{\varepsilon_m(1-\nu_m)}{1-\varepsilon_m(1-\nu_m)}}$. TFP in the net production function Y_m, $\Phi_{m1} B_m^{\frac{1}{1-\varepsilon_m(1-\nu_m)}}$, is a function of gross output TFP, B_m, and of the elasticity of manufacturing gross output with respect to manufactured intermediate goods, $\varepsilon_m (1-\nu_m)$.[1] Equation (2) can be re-written as

$$Y_m = A_m \left(K_m^{\alpha} N_m^{1-\alpha} \right)^{\theta} S^{1-\theta}, \tag{3}$$

where

$$A_m = \Phi_{m1} B_m^{\frac{1}{1-\varepsilon_m(1-\nu_m)}}, \tag{4}$$

$0 < \theta < 1$ is equal to $\frac{\nu_m}{1-\varepsilon_m(1-\nu_m)}$ and $S_m = S$. The problem of the firm in the manufacturing sector becomes

$$\max_{K_m, N_m, S} \left[p_m Y_m - r K_m - w N_m - p_s S \right] \tag{5}$$

subject to (3).

The net production function in the services sector is accordingly derived and it is given by

$$Y_s = A_s \left[(K_s)^{\alpha} (N_s)^{1-\alpha} \right]^{\gamma} M^{1-\gamma}, \tag{6}$$

[1] Note that the price of G_m and Y_m is the same.

where $0 < \gamma < 1$ is equal to $\frac{\nu_s}{1-\varepsilon_s(1-\nu_s)}$, K_s and N_s are the amounts of capital and labor and M is the amount of manufacturing used as intermediate good in the services sector. Finally,

$$A_s = \Phi_{s1} B_s^{\frac{1}{1-\varepsilon_s(1-\nu_s)}},$$

with $\Phi_{s1} = [1-\varepsilon_s(1-\nu_s)] [\varepsilon_s(1-\nu_s)]^{\frac{\varepsilon_s(1-\nu_s)}{1-\varepsilon_s(1-\nu_s)}}$.

The problem of the representative firm in services becomes

$$\max_{K_s, N_s, M} [p_s Y_s - rK_s - wN_s - p_m M] \tag{7}$$

subject to (6).

With competitive markets each firm sets the price equal to the marginal cost. Given the Cobb-Douglas form of (3) and (6), the price of the manufacturing good is

$$p_m = \frac{\left(r^\alpha w^{1-\alpha}\right)^\theta p_s^{1-\theta}}{\Phi_{m2} A_m}, \tag{8}$$

and that of services

$$p_s = \frac{\left(r^\alpha w^{1-\alpha}\right)^\gamma p_m^{1-\gamma}}{\Phi_{s2} A_s}, \tag{9}$$

where Φ_{m2} is a function of α and θ and Φ_{s2} is a function of α and γ.[2] By solving (8) and (9) for p_m and p_s it is possible to write

$$\frac{p_s}{p_m} = \frac{\left(\Phi_{m2} A_m\right)^{\frac{\gamma}{\gamma+\theta-\theta\gamma}}}{\left(\Phi_{s2} A_s\right)^{\frac{\theta}{\gamma+\theta-\theta\gamma}}}. \tag{10}$$

By substituting the definition of A_m, A_s, θ and γ in (10)

$$\frac{p_s}{p_m} = \frac{\left(\Phi_{m1}\Phi_{m2} B_m^{\frac{1}{1-\varepsilon_m(1-\nu_m)}}\right)^{\frac{\nu_s[1-\varepsilon_m(1-\nu_m)]}{\nu_m[1-\varepsilon_s(1-\nu_s)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_s\nu_m}}}{\left(\Phi_{s1}\Phi_{s2} B_s^{\frac{1}{1-\varepsilon_s(1-\nu_s)}}\right)^{\frac{\nu_m[1-\varepsilon_s(1-\nu_s)]}{\nu_m[1-\varepsilon_s(1-\nu_s)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_s\nu_m}}}, \tag{11}$$

where now $\Phi_{m2} = [\alpha^\alpha(1-\alpha)^{1-\alpha}]^{\frac{\nu_m}{1-\varepsilon_m(1-\nu_m)}} \left(\frac{\nu_m}{1-\varepsilon_m(1-\nu_m)}\right)^{\frac{\nu_m}{1-\varepsilon_m(1-\nu_m)}} \left(\frac{(1-\varepsilon_m)(1-\nu_m)}{1-\varepsilon_m(1-\nu_m)}\right)^{\frac{(1-\varepsilon_m)(1-\nu_m)}{1-\varepsilon_m(1-\nu_m)}}$ and $\Phi_{s2} = [\alpha^\alpha(1-\alpha)^{1-\alpha}]^{\frac{\nu_s}{1-\varepsilon_s(1-\nu_s)}} \left(\frac{\nu_s}{1-\varepsilon_s(1-\nu_s)}\right)^{\frac{\nu_s}{1-\varepsilon_s(1-\nu_s)}} \left(\frac{(1-\varepsilon_s)(1-\nu_s)}{1-\varepsilon_s(1-\nu_s)}\right)^{\frac{(1-\varepsilon_s)(1-\nu_s)}{1-\varepsilon_s(1-\nu_s)}}$. Equation (11) can be simplified to obtain the relative price of the two goods

$$\frac{p_s}{p_m} = \Omega \left(\frac{B_m^{\nu_s}}{B_s^{\nu_m}}\right)^{\frac{1}{\nu_m[1-\varepsilon_s(1-\nu_s)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_s\nu_m}}, \tag{12}$$

[2] $\Phi_{m2} = (\alpha\theta)^{\alpha\theta} [(1-\alpha)\theta]^{(1-\alpha)\theta} (1-\theta)^{1-\theta}$ and $\Phi_{s2} = (\alpha\gamma)^{\alpha\gamma} [(1-\alpha)\gamma]^{(1-\alpha)\gamma} (1-\gamma)^{1-\gamma}$.

where

$$\Omega = \frac{\left(\Phi_{m1}\Phi_{m2}\right)^{\frac{\nu_s[1-\varepsilon_m(1-\nu_m)]}{\nu_m[1-\varepsilon_s(1-\nu_s)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_s\nu_m}}}{\left(\Phi_{s1}\Phi_{s2}\right)^{\frac{\nu_m[1-\varepsilon_s(1-\nu_s)]}{\nu_m[1-\varepsilon_s(1-\nu_s)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_s\nu_m}}}.$$

To find the aggregate production in manufacturing units, equation (5), it is convenient to use again the net production functions (3) and (6). The aggregate production function is the solution to the following problem

$$\max_{K_m,N_m,M,S}\left[A_m\left(K_m^{\alpha}N_m^{1-\alpha}\right)^{\theta}S^{1-\theta}-M\right] \tag{13}$$

subject to

$$A_s\left[(K-K_m)^{\alpha}(N-N_m)^{1-\alpha}\right]^{\gamma}M^{1-\gamma}=S,$$

where K and N are the aggregate capital and labor available for production in the period considered. The solution to this problem is

$$V_m=\Phi_{m3}A_m^{\frac{1}{\gamma+\theta-\theta\gamma}}A_s^{\frac{1-\theta}{\gamma+\theta-\theta\gamma}}K^{\alpha}N^{1-\alpha}, \tag{14}$$

where $\Phi_{m3}=[1-(1-\theta)(1-\gamma)][(1-\theta)(1-\gamma)]^{\frac{1-\theta-\gamma+\theta\gamma}{\theta+\gamma-\theta\gamma}}\left(\frac{\theta}{\gamma+\theta-\gamma\theta}\right)^{\frac{\theta}{\theta+\gamma-\theta\gamma}}\left(\frac{\gamma.(1-\theta)}{\gamma+\theta-\gamma\theta}\right)^{\frac{\gamma(1-\theta)}{\theta+\gamma-\theta\gamma}}$. By substituting the definitions $A_m=\Phi_{m1}B_m^{\frac{1}{1-\varepsilon_m(1-\nu_m)}}$ and $A_s=\Phi_{s1}B_s^{\frac{1}{1-\varepsilon_s(1-\nu_s)}}$, (14) becomes

$$V_m=\Phi_{m1}^{\frac{1}{\gamma+\theta-\theta\gamma}}\Phi_{s1}^{\frac{1-\theta}{\gamma+\theta-\theta\gamma}}\Phi_{m3}B_m^{\frac{1}{1-\varepsilon_m(1-\nu_m)}\frac{1}{\gamma+\theta-\theta\gamma}}B_s^{\frac{1}{1-\varepsilon_s(1-\nu_s)}\frac{1-\theta}{\gamma+\theta-\theta\gamma}}K^{\alpha}N^{1-\alpha}. \tag{15}$$

By defining $\Theta_m=\Phi_{m1}^{\frac{1}{\gamma+\theta-\theta\gamma}}\Phi_{s1}^{\frac{1-\theta}{\gamma+\theta-\theta\gamma}}\Phi_{m3}$ and using the definitions $\theta=\frac{\nu_m}{1-\varepsilon_m(1-\nu_m)}$ and $\gamma=\frac{\nu_s}{1-\varepsilon_s(1-\nu_s)}$ it is possible to write (15) as

$$V_m=\Theta_m B_m^{f_1}B_s^{f_2}K^{\alpha}N^{1-\alpha}, \tag{16}$$

which is equation (5) in the paper. In (16) $f_1=\frac{1-\varepsilon_s(1-\nu_s)}{\nu_m[1-\varepsilon_s(1-\nu_s)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_m\nu_s}$ and $f_2=\frac{(1-\varepsilon_m)(1-\nu_m)}{\nu_m[1-\varepsilon_s(1-\nu_s)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_m\nu_s}$. By dividing (16) by (12) it is possible to obtain

$$V_s=\Theta_s B_m^{f_3}B_s^{f_4}K^{\alpha}N^{1-\alpha},$$

$\Theta_s=\Theta_m/\Omega$, $f_3=\frac{(1-\varepsilon_s)(1-\nu_s)}{\nu_m[1-\varepsilon_s(1-\nu_s)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_m\nu_s}$ and $f_4=\frac{1-\varepsilon_m(1-\nu_m)}{\nu_m[1-\varepsilon_s(1-\nu_s)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_m\nu_s}$, which is equation (6) in the text.

Finally, to prove that $f_1+f_2>f_3+f_4$ if and only if $\nu_m<\nu_s$, note that the denominator of f_1, f_2, f_3 and f_4 is always positive. This is evident by re-writing it as $\nu_m(1-\varepsilon_s)+\nu_s(1-\varepsilon_m)+\nu_m\nu_s\varepsilon_s+\nu_m\nu_s\varepsilon_m$. Thus, by using the numerators, $f_1+f_2>f_3+f_4$ reads

$$1-\varepsilon_s(1-\nu_s)+(1-\varepsilon_m)(1-\nu_m)>(1-\varepsilon_s)(1-\nu_s)+1-\varepsilon_m(1-\nu_m)$$

which can be simplified to obtain $\nu_m<\nu_s$.